Pre-calculus 11 Sample Final Assessment Questions Scoring Guide

Note Title 5/18/2015

1) 50, 42.5, 36.125, ..., ???

at start t_0; after 24 hrs (1 day) t_1; after 48 hrs (2 days) t_2; after 120 hrs (5 days) t_5

$t_1 = 42.5$
$r = 0.85$
$n = 5$
$t_5 = ??$

$t_n = t_1 r^{n-1}$
$t_5 = 42.5(0.85)^{5-1}$
$= 42.5(0.85)^4$
$t_5 = 22.2$ mg

OR

50, 42.5, 36.125, ..., ???

at start of day 1 t_1; after 24 hrs (start of day 2) t_2; after 48 hrs (start of day 3) t_3; after 120 hrs (start of day 6) t_6

$t_1 = 50$
$r = 0.85$
$n = 6$
$t_6 = ??$

$t_n = t_1 r^{n-1}$
$t_6 = 50(0.85)^{6-1}$
$t_6 = 50(0.85)^5$
$t_6 = 22.2$ mg

Scoring

1 pt: ratio of 0.85
1 pt: correct t_1 and r
½ pt: substituting correctly into formula
½ pt: final answer
* Subtract ½ pt if correct units are not included with answer

2) $11-x$, $2x+1$ and $3x+1$

$(2x+1)-(11-x) = (3x+1)-(2x+1)$
$3x-10 = x$
$2x = 10$
$\boxed{x=5}$

$\therefore t_1 = 11-x = 6$
$t_2 = 2x+1 = 11$
$t_3 = 3x+1 = 16$
$\boxed{d=5}$

OR

$t_1 = 11-x$
$t_2 = (11-x)+d = 2x+1$
$t_3 = (11-x)+2d = 3x+1$
$\therefore 11-x+d = 2x+1$ and $11-x+2d = 3x+1$
$d = 3x-10$ $\quad 2d = 4x-10$
or $d = 2x-5$
$\therefore 2x-5 = 3x-10$
$\boxed{5 = x}$ and $d = 2x-5$
$d = 2(5)-5$
$\boxed{d=5}$

$S_n = \frac{n}{2}(2t_1+(n-1)d)$
$S_{15} = \frac{15}{2}(2(6)+(14)(5))$

$S_{15} = 7.5(12+70) = 615$

Scoring:

1 pt: $t_2 - t_1 = t_3 - t_2$
1 pt: Solving for x
1 pt: using x value to determine d
½ pt: filling in correctly to S_n formula
½ pt: final answer

Scoring:

1 pt: $t_2 = t_1 + d$ and $t_3 = t_1 + 2d$
1 pt: solving for x
1 pt: using x value to determine d
½ pt: filling in correctly to S_n formula
½ pt: final answer

3) $t_3 = x - 2$

$t_4 = x + 1$

$t_5 = x + 7$

$\frac{t_4}{t_3} = \frac{t_5}{t_4}$ or $t_4 = r t_3$ and $t_5 = r t_4$ $\therefore \frac{t_4}{t_3} = \frac{t_5}{t_4}$

$\frac{x+1}{x-2} = \frac{x+7}{x+1}$

$(x+1)(x+1) = (x-2)(x+7)$

$x^2 + 2x + 1 = x^2 + 5x - 14$

$2x + 1 = 5x - 14$

$15 = 3x$

$x = 5$

$\left.\begin{array}{l} t_3 = x - 2 = 3 \\ t_4 = x + 1 = 6 \\ t_5 = x + 7 = 12 \end{array}\right] r = 2$

$\frac{3}{4}, \frac{3}{2}, 3, 6, 12$

$\therefore t_1 = \frac{3}{4}$; $t_2 = \frac{3}{2}$

OR

$t_3 = t_1 r^2$

$3 = t_1 (2)^2$

$\frac{3}{4} = t_1$

$t_2 = t_1 (r)$

$= (\frac{3}{4})(2)$

$t_2 = \frac{3}{2}$

Scoring:

1 pt: $\frac{t_4}{t_3} = \frac{t_5}{t_4}$

1 pt: solving for x

1 pt: using value of x to determine r

$\frac{1}{2}$ pt: using r and term given to obtain t_1

$\frac{1}{2}$ pt: determining t_2.

4) $t_5 + t_{12} = -53 \rightarrow (t_1 + 4d) + (t_1 + 11d) = -53$

$t_{10} + t_{100} = -332 \rightarrow (t_1 + 9d) + (t_1 + 99d) = -332$

$2t_1 + 15d = -53$

$\ominus\ 2t + 108d = -332$

$-93d = 279$

$d = -3$

$2t_1 + 15d = -53$

$2t_1 - 45 = -53$

$2t_1 = -8$

$t_1 = -4$

$S_n = \frac{n}{2}(2t_1 + (n-1)d)$

$S_{50} = \frac{50}{2}(2(-4) + (49)(-3))$

$= 25(-8 - 147)$

$S_{50} = -3875.$

Scoring:

1 pt: replacing each term by $t_1 + (n-1)d$

$\frac{1}{2}$ pt: obtaining system of eqns

1 pt: solving system of equations for d

$\frac{1}{2}$ pt: using value of d to obtain t_1

$\frac{1}{2}$ pt: substituting correctly into S_n formula

$\frac{1}{2}$ pt: final answer

5) $\sin 45^\circ = \frac{\sqrt{2}}{2}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\cos 0^\circ = 1$

$\cos 135^\circ = -\frac{\sqrt{2}}{2}$

$\sin 135^\circ = \frac{\sqrt{2}}{2}$

$$\frac{\sin 45^\circ}{\sin 60^\circ} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{2}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

$\therefore$ Numerator

$$\frac{\sin 45^\circ}{\sin 60^\circ} + \cos 30^\circ = \frac{\sqrt{6}}{3} + \frac{\sqrt{3}}{2}$$

$$= \frac{2\sqrt{6} + 3\sqrt{3}}{6}$$

$$\frac{\cos 0^\circ}{\cos 135^\circ} = \frac{1}{-\sqrt{2}/2}$$

$$= \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$$

$\therefore$ Denominator

$$\frac{\cos 0^\circ}{\cos 135^\circ} + \sin 135^\circ$$

$$-\sqrt{2} + \frac{\sqrt{2}}{2} = \frac{-2\sqrt{2} + \sqrt{2}}{2} = \frac{-\sqrt{2}}{2}$$

Final Value:

$$\frac{\text{Numerator}}{\text{Denominator}} = \frac{\frac{2\sqrt{6}+3\sqrt{3}}{6}}{\frac{-\sqrt{2}}{2}}$$

$$= \frac{2\sqrt{6}+3\sqrt{3}}{6} \cdot \frac{2}{-\sqrt{2}}$$

$$= \frac{2\sqrt{6}+3\sqrt{3}}{-3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{2\sqrt{12} + 3\sqrt{6}}{-6}$$

$$= \frac{4\sqrt{3} + 3\sqrt{6}}{-6}$$

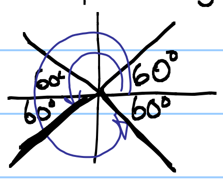
or $-\frac{2}{3}\sqrt{3} - \frac{1}{2}\sqrt{6}$

Scoring

- $\frac{1}{2}$ pt exact value of $\sin 45^\circ$
- $\frac{1}{2}$ pt exact value of $\sin 60^\circ$
- $\frac{1}{2}$ pt exact value of $\cos 30^\circ$
- $\frac{1}{2}$ pt exact value of $\cos 135^\circ$
- $\frac{1}{2}$ pt exact value of $\sin 135^\circ$
- $\frac{1}{2}$ pt rationalizing ($\sin 45^\circ / \sin 60^\circ$)
- $\frac{1}{2}$ pt simplifying numerator
- $\frac{1}{2}$ pt rationalizing ($\cos 0^\circ / \cos 135^\circ$)
- $\frac{1}{2}$ pt simplifying denominator
- $\frac{1}{2}$ pt rationalizing fraction
- 1 pt simplifying final answer.

6) $\sin\theta = -\frac{\sqrt{3}}{2}$

principle angle for $\sin\theta = \frac{\sqrt{3}}{2}$ is 60°



sine is negative in quadrants III and IV

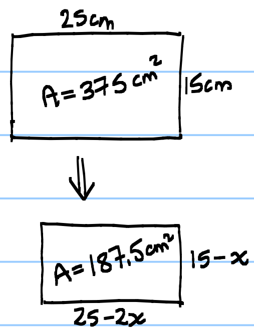
$\therefore \theta = 180^\circ + 60^\circ$ and $360^\circ - 60^\circ$

$\theta = 240^\circ$ and $\theta = 300^\circ$

Scoring:

- 1 pt for recognizing angles must be in III and IV quadrants
- 1 pt for principle angle
- $\frac{1}{2}$ pt for $180^\circ +$ principle
- $\frac{1}{2}$ pt for $360^\circ -$ principle

7)



Scoring:

½ pt: width of (15−x)

½ pt: length of (25−2x)

½ pt: area for new rect. of ½ Area of original

1 pt: setting up equation
(width of new)(length of new) = Area of new

½ pt: obtaining quadratic

½ pt: substituting correctly into quadratic formula

½ pt: solving for x

½ pt: eliminating extraneous solution

½ pt: using value of x to correctly determine new dimensions

$\therefore (15-x)(25-2x) = 187.5$

$375 - 30x - 25x + 2x^2 = 187.5$

$2x^2 - 55x + 187.5 = 0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{55 \pm \sqrt{3025 - 4(2)(187.5)}}{4}$

$x = \dfrac{55 \pm \sqrt{1525}}{4} = \dfrac{55 \pm 39.05}{4}$ ↗ ~~23.5 cm~~ ↘ 4.0 cm (approx.)

∴ New dimensions are

$15 - x \doteq 11$ cm

$25 - 2x \doteq 17$ cm

8a) $h(t) = -5t^2 + 40t + 1$

$h(t) = -5(t^2 - 8t) + 1$

$h(t) = -5(t^2 - 8t + 16) + 1 + 5(16)$

$h(t) = -5(t-4)^2 + 81$

Scoring:

½ pt for factoring out −5

½ pt for completing square

½ pt for adding 5(16)

8 b) $h(t) = -5t^2 + 40t + 1$

We know at t=0 sec the ball is 1 m high

$h(t) = -5(t-4)^2 + 81$

We know the ball reaches a max height of 81 m. after 4 sec.

Scoring:

1 pt: At t=0 s. h(t) = 1 m.

1 pt: At t=4 s h(t) = max of 81 m

8c) Since it took 4 seconds to reach its max height we know (because of symmetry) it will take 4 more seconds to return to its original height. ∴ After 8 s. the ball will return to a height of 1 m.

Scoring:

1 pt: Use of symmetry

½ pt: Answer

9) $3x^2 + 6x + 1 = y$

$3(x^2 + 2x + 1) + 1 - 3(1) = y$

$3(x+1)^2 - 2 = y$

$\therefore$ x-intercepts when $y = 0$

$3(x+1)^2 - 2 = 0$

$3(x+1)^2 = 2$

$(x+1)^2 = 2/3$

$x + 1 = \pm \sqrt{2}/\sqrt{3} \cdot \sqrt{3}/\sqrt{3}$

$x + 1 = \pm \frac{\sqrt{6}}{3}$

$x = -1 \pm \sqrt{6}/3$

OR $x = \frac{-3 \pm \sqrt{6}}{3}$

Scoring:

½ pt : factoring out 3

½ pt : completing the square

½ pt : Quadratic in vertex form

½ pt : $y = 0$

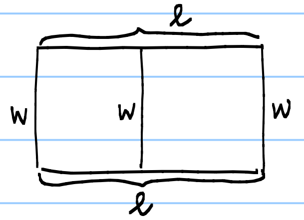
½ pt : Isolating squared term

½ pt : square root [must include ±]

½ pt : rationalizing answer

½ pt : final answer

10)



$3w + 2l = 210 \rightarrow 2l = 210 - 3w$ or $3w = 210 - 2l$

$l = 105 - 1.5w$ | $w = 70 - \frac{2}{3}l$

$w(l) = A$

$\therefore w(105 - 1.5w) = A$

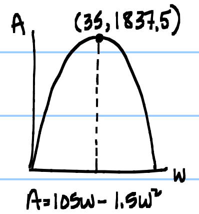
$105w - 1.5w^2 = A$

$w = -\frac{b}{2a}$ or Graphically:

$w = \frac{-105}{2(-1.5)}$

$= \frac{-105}{3}$

$\boxed{w = 35\ m}$



$l = 105 - 1.5(w)$

$\boxed{l = 52.5m}$

$(70 - \frac{2}{3}l)(l) = A$

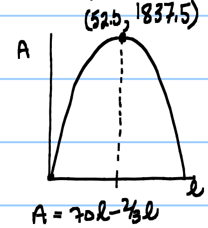
$70l - \frac{2}{3}l^2 = A$

$l = -\frac{b}{2a}$ or Graphically:

$l = \frac{-70}{2(-\frac{2}{3})}$

$l = -70/\frac{4}{3}$

$\boxed{l = 52.5m}$



$w = 70 - \frac{2}{3}(l)$

$\boxed{w = 35m}$

Scoring

½ pt : Adding $3w + 2l$

½ pt : Sum of fencing = 210m

½ pt : Solving equation for l or w

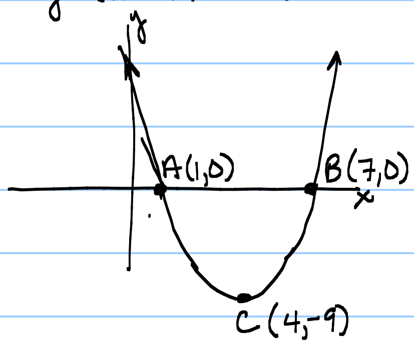
1 pt : substituting into $A = lw$

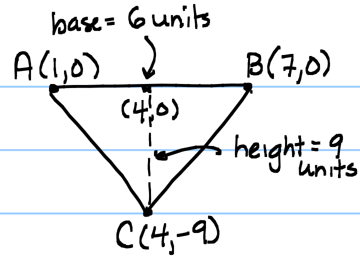
1 pt : determining l or w that creates maximum area

½ pt : Substituting to determine other dimension

11) $y = x^2 - 8x + 7$

$y = (x-7)(x-1)$





$\therefore A = \frac{1}{2}bh$

$= \frac{1}{2}(6)(9)$

$= 27 \text{ units}^2$

Scoring

1pt: obtaining the x-intercepts

1pt: obtaining the vertex

$\frac{1}{2}$pt: length of base of Δ

1 pt: height of Δ

$\frac{1}{2}$pt: area of Δ

$y = (x^2 - 8x + 16) + 7 - 16$

$y = (x-4)^2 - 9$

12) $\sqrt{5-x} - 3 = x + 4$

$\sqrt{5-x} = x + 7$

$5 - x = x^2 + 14x + 49$

$0 = x^2 + 15x + 44$

$0 = (x+11)(x+4)$

$\cancel{x = -11}$ $\qquad$ $x = -4$

Check $\qquad$ Check

$\sqrt{16} - 3 \neq -11 + 4$ $\qquad$ $\sqrt{9} - 3 = -4 + 4$ ✓

Scoring:

$\frac{1}{2}$pt: Isolating square root

1 pt: correctly squaring both sides

1 pt: solving resulting quadratic

$\frac{1}{2}$pt: verifying solution(s)

13) $f(x) = 2x^2 - 3x + 4$

$f(5+2\sqrt{3}) = 2(5+2\sqrt{3})^2 - 3(5+2\sqrt{3}) + 4$

$= 2(25 + 20\sqrt{3} + 12) - 15 - 6\sqrt{3} + 4$

$= 50 + 40\sqrt{3} + 24 - 15 - 6\sqrt{3} + 4$

$= 63 + 34\sqrt{3}$

Scoring:

1pt: substiting $5+2\sqrt{3}$ for x

$\frac{1}{2}$pt squaring $(5+2\sqrt{3})$ correctly

$\frac{1}{2}$pt expanding correctly

1pt combining like terms to obtain final answer

14) $$\frac{(3\sqrt{5m})(2\sqrt{5})}{2\sqrt{m}-4}$$

$$= \frac{6\sqrt{25m}}{2\sqrt{m}-4}$$

$$= \frac{30\sqrt{m}}{2\sqrt{m}-4} \cdot \frac{2\sqrt{m}+4}{2\sqrt{m}+4}$$

$$= \frac{60m+120\sqrt{m}}{4m-16}$$

$$= \frac{4(15m+30\sqrt{m})}{4(m-4)}$$

$$= \frac{15m+30\sqrt{m}}{m-4} \quad \text{or} \quad \frac{15(m+2\sqrt{m})}{m-4}$$

Scoring:

½ pt: mult terms in numerator correctly
½ pt: Simplifying numerator
1pt: mult. by conjugate of denominator
1pt: rationalizing and simplifying

15) $$2\sqrt{x^2-3x-9}+1 = 2x-3$$

$$2\sqrt{x^2-3x-9} = 2x-4$$

$$\sqrt{x^2-3x-9} = x-2$$

$$x^2-3x-9 = x^2-4x+4$$

$$-3x-9 = -4x+4$$

$$x = 13$$

Check:

$$2\sqrt{169-39-9}+1 = 26-3$$

$$2\sqrt{121}+1 = 23 \quad ✓$$

Scoring:

1pt: Isolate square root term
1pt: square both sides
½ pt: solve for x
½ pt: check solution

16) $$\frac{x^2-4x-12}{x+3} + \frac{x^2-7x}{x-7} = \frac{5x+10}{x+3} + \frac{2x^2-14x}{2x-14}$$

$$\frac{(x-6)(x+2)}{x+3} + \frac{x\cancel{(x-7)}}{\cancel{x-7}} = \frac{5(x+2)}{x+3} + \frac{\cancel{2}x\cancel{(x-7)}}{\cancel{2}\cancel{(x-7)}}$$

$$\frac{(x-6)(x+2)}{x+3} + \cancel{x} = \frac{5(x+2)}{x+3} + \cancel{x}$$

$$\frac{(x-6)(x+2)}{x+3} = \frac{5(x+2)}{x+3}$$

Equating the numerators:

$$(x-6)(x+2) = 5(x+2)$$

$$x^2-4x-12 = 5x+10$$

$$x^2-9x-22=0; \ \boxed{x=2, x=11}$$

OR

$$\frac{(x-6)(x+2)+x(x+3)}{x+3} = \frac{5(x+2)+x(x+3)}{x+3}$$

$$\frac{x^2-4x-12+x^2+3x}{x+3} = \frac{5x+10+x^2+3x}{x+3}$$

$$\frac{2x^2-x-12}{x+3} = \frac{x^2+8x+10}{x+3}$$

$$2x^2-x-12 = x^2+8x+10$$

$$x^2-9x-22=0$$

$$(x-11)(x+2)=0$$

$$\boxed{x=11} \quad \boxed{x=-2}$$

Scoring:

2pts: Factoring
1pt: Reducing fractions
1pt: Equating numerators
1pt: Final answer

17a) $\dfrac{x^2+3x-10}{x^3+6x^2+5x} \div \dfrac{x^2-3x}{x^2+5x+4}$

Scoring:
½ pt: mult by reciprocal
1 pt: factoring
1 pt: reducing/simplifying
½ pt: stating restrictions

$\dfrac{x^2+3x-10}{x^3+6x^2+5x} \cdot \dfrac{x^2+5x+4}{x^2-3x}$

$\dfrac{\cancel{(x+5)}(x-2)}{x\cancel{(x+5)}\cancel{(x+1)}} \cdot \dfrac{(x+4)\cancel{(x+1)}}{x(x-3)}$

$\dfrac{(x-2)(x+4)}{x^2(x-3)}$; $x \neq -5, -4, -1, 0, 3$

17b) $\dfrac{\frac{1+x}{x} - 2}{2 + \frac{1-x}{x}} = \dfrac{\frac{1+x-2(x)}{x}}{\frac{2(x)+1-x}{x}}$

Scoring:
1pt: getting common denominator
½pt: simplifying
½pt: mult by reciprocal
½pt: simplifying
½pt: stating restrictions

$= \dfrac{\frac{1-x}{x}}{\frac{x+1}{x}} = \dfrac{1-x}{x} \cdot \dfrac{x}{x+1}$

$= \dfrac{1-x}{x+1}$; $x \neq 0, x \neq -1$

17c) $\dfrac{2x^2-5x-12}{x^2-2x-8} - \dfrac{x^2+4x+8}{x^2-4}$

Scoring:
1pt: factoring
1pt: common denominator
1½pts: simplifying
½pt: stating restrictions

$\dfrac{(2x+3)\cancel{(x-4)}}{\cancel{(x-4)}(x+2)} - \dfrac{x^2+4x+8}{(x+2)(x-2)}$

$\dfrac{(2x+3)(x-2) - x^2 - 4x - 8}{(x+2)(x-2)}$

$\dfrac{2x^2-4x+3x-6-x^2-4x-8}{(x+2)(x-2)}$

$\dfrac{x^2-5x-14}{(x+2)(x-2)} = \dfrac{(x-7)\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}$

$\dfrac{x-7}{x-2}$; $x \neq \pm 2, x \neq 4$

18) $\frac{7x}{3x+4} - \frac{28x}{2x+5} = -3$

$\frac{(7x)(2x+5) - (28x)(3x+4)}{(3x+4)(2x+5)} = -3$

$\frac{14x^2 + 35x - 84x^2 - 112x}{(3x+4)(2x+5)} = -3$

$\frac{-70x^2 - 77x}{6x^2 + 15x + 8x + 20} = -3$

$-70x^2 - 77x = -3(6x^2 + 23x + 20)$

$-70x^2 - 77x = -18x^2 - 69x - 60$

$0 = 52x^2 + 8x - 60$

$0 = 13x^2 + 2x - 15$

$0 = (13x + 15)(x - 1)$

$x = -15/13 \qquad x = 1$

Scoring

1 pt: common denominator

1 pt: simplifying LHS

1 pt: obtaining quadratic

1 pt: solving quadratic

19) $\frac{x^3 + 2x^2}{x^2 - x - 2} - (x-2)$

$= \frac{x^2(x-2)}{(x-2)(x+1)} - (x-2)$

$= \frac{x^2}{x+1} - \frac{(x-2)(x+1)}{x+1}$

$= \frac{x^2 - (x^2 - x - 2)}{x+1}$

$= \frac{x+2}{x-1}$

Scoring:

1 pt: factoring

1 pt: common denominator

1 pt: algebra to obtain answer

20)

	distance (m)	speed (m/s)	time (sec)
last year	600	x	t
this year	600	$x+3$	$t-10$

$\frac{d}{t} = \text{Speed}$ or $d = (\text{speed})(\text{time})$

$\therefore 600 = (x)(t) \rightsquigarrow x = \frac{600}{t}$

$600 = (x+3)(t-10)$

$600 = \left(\frac{600}{t} + 3\right)(t - 10)$

$600 = 600 + 3t - \frac{6000}{t} - 30$

$30 = 3t - \frac{6000}{t} \Rightarrow 30t = 3t^2 - 6000$

$3t^2 - 30t - 6000 = 0$

$t^2 - 10t - 2000 = 0$

$t = \frac{10 \pm \sqrt{100 - 4(1)(-2000)}}{2}$

$t = \frac{10 \pm \sqrt{8100}}{2} = \frac{10 \pm 90}{2} \nearrow 50, \searrow -40$

$\therefore$ time last year was 50 seconds

Scoring:

1 pt: obtaining equation for last year

1 pt: obtaining equation for this year

1 pt: Isolating variable and substituting

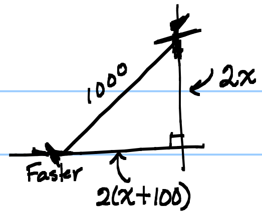
$1\frac{1}{2}$ pts: Solving for t

$\frac{1}{2}$ pt: final answer (including units)

21)

	Distance (km)	Speed (km/h)	Time (h)
Slower plane	$2x$	x	2
Faster plane	$2(x+100)$	$x+100$	2

$\uparrow$
$d=(v)(t)$



Using Pythagorean Theorem:

$(2x)^2 + (2(x+100))^2 = (1000)^2$

$4x^2 + 4(x^2+200x+10000) = 1000000$

$4x^2 + 4x^2 + 800x + 40000 = 1000000$

$8x^2 + 800x - 960000 = 0$

$x^2 + 100x - 120000 = 0$

$x = \dfrac{-100 \pm \sqrt{10000-4(1)(-120000)}}{2}$

$= \dfrac{-100 \pm \sqrt{490000}}{2}$

$x = \dfrac{-100 \pm 700}{2}$ → 300 km/h, −400 km/h

∴ The slower plane flies 300 km/h
The faster plane flies 400 km/h.

Scoring

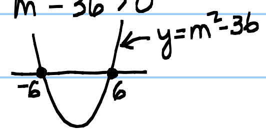
$\frac{1}{2}$ pt: expression for dist. of slower plane
1 pt: expression for dist. of faster plane
1 pt: using pythagorean theorem
1 pt: expanding correctly to get quadratic
1 pt: Solving quadratic for x
$\frac{1}{2}$ pt: final answers (including units)

22) $x^2 + mx + 9 = 0$

$b^2 - 4ac > 0$

$m^2 - 4(1)(9) > 0$

$m^2 - 36 > 0$



∴ $y>0$ when $m<-6$; $m>6$

Scoring

1 pt: setting discriminant >0
1 pt: sketching $y(m) = m^2-36$ to determine when $m^2-36>0$
1 pt: final answer

23)
$5x^2 - 3x + 9 - y = 10$
$\ominus\ 6x^2 + 2x - 13 - y = 2$
$-x^2 - 5x + 22 = 8$

OR

$5x^2 - 3x - 1 = y$
$6x^2 + 2x - 13 - [5x^2 - 3x - 1] = 2$
$x^2 + 5x - 12 = 2$

$0 = x^2 + 5x - 14$
$0 = (x+7)(x-2)$
$x = -7$; $x = 2$

∴ Pts are
$(-7, 265)$
$(2, 13)$

Scoring:

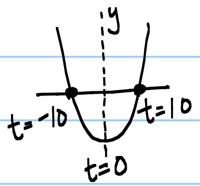
1 pt isolating and substituting or using elimination
1 pt solving for values of x
1 pt determining y-values of coordinates

24a) $-0.2t^2+30 \le P$

$-0.2t^2+30 \le 10$

$-0.2t^2+20 \le 0$

$t^2-100 \geqslant 0$



$\therefore [0,10]$

Scoring:

$\frac{1}{2}$pt: $P=10$

1pt: obtaining $t^2-100 \geqslant 0$

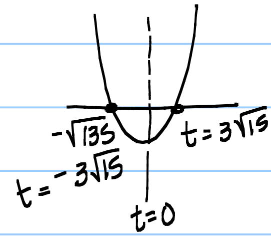
1pt: solving t^2-100

$\frac{1}{2}$pt: final answer

24 b) $-0.2t^2+30 \le 3$

$-0.2t^2+27 \le 0$

$t^2-135 \le 0$



$\therefore [0, 3\sqrt{15}]$

Scoring:

$\frac{1}{2}$pt: $P=3$

1pt: obtaining $t^2-135 \geqslant 0$

1pt: solving t^2-135

$\frac{1}{2}$pt: final answer

24c) In context we consider the function to be valid only for $t \geqslant 0$.

Scoring:

1pt: context mentioned (restricted domain)

25 a) $y = |3x+1|$

x-int: $|3x+1| = 0$
(y=0)

$3x+1=0$

$3x=-1$

$x=-\frac{1}{3}$

$\left.\right\} (-\frac{1}{3}, 0)$

y-int: $y = |3(0)+1|$
(x=0)

$y=1$

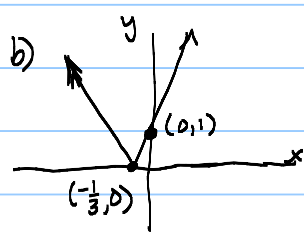
$\left.\right\} (0,1)$

Scoring:

$\frac{1}{2}$pt: y-intercept

$\frac{1}{2}$pt: x-intercept

25 b)



Scoring

1pt: sketch

(shape and intercepts must be correct)

25c) Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \geqslant 0, y \in \mathbb{R}\}$

Scoring:

$\frac{1}{2}$ pt: domain

$\frac{1}{2}$ pt: range

24d) $y = \begin{cases} -(3x+1) \,; & x \le -\frac{1}{3} \\ 3x+1 \,; & x > -\frac{1}{3} \end{cases}$

Scoring:

$\frac{1}{2}$pt: left branch

$\frac{1}{2}$pt: right branch

26 a) $y = |x^2 - x - 2|$

x-int: $|x^2-x-2| = 0$ $(y=0)$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = +2 \,;\, x = -1$

$\left.\right\}$ $(-1,0)$, $(2,0)$

y-int: $y = |0^2 - 0 - 2|$ $(x=0)$

$y = |-2| = 2$

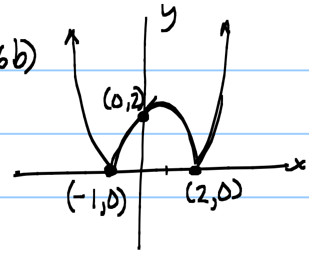
$\left.\right\}$ $(0,2)$

Scoring:

½ pt: y-intercepts

½ pt: x-intercept

26b)



Scoring

1 pt: sketch

(shape and intercepts must be correct)

½ pt: sketch's vertex at $x = \frac{1}{2}$ (above and to the right of (0,2))

26c) Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

Scoring:

½ pt: domain

½ pt: range

24d) $y = \begin{cases} x^2 - x - 2 \,; & x \leq -1 \\ -(x^2 - x - 2 \,; & -1 < x < 2 \\ x^2 - x - 2 \,; & x \geq 2 \end{cases}$

Scoring:

½ pt: left branch

½ pt: middle section

½ pt: right branch

27 a) $|x+5| = 4x - 1$

$x + 5 > 0$	$x + 5 < 0$
$x + 5 = 4x - 1$	$-(x+5) = 4x - 1$
$6 = 3x$	$x + 5 = -4x + 1$
$2 = x$	$5x = -4$
	$\cancel{x = -4/5}$

Scoring

1 pt solving when $x + 5 > 0$

1 pt solving when $x + 5 \leq 0$

1 pt: eliminating incorrect answers

27 b) $|x-5| = x^2 - 8x + 15$

$x - 5 \geq 0$	$x - 5 \leq 0$
$x - 5 = x^2 - 8x + 15$	$-(x-5) = x^2 - 8x + 15$
$0 = x^2 - 9x + 20$	$-x + 5 = x^2 - 8x + 15$
$0 = (x-5)(x-4)$	$0 = x^2 - 7x + 10$
$x = 5$, $\cancel{x = 4}$	$0 = (x-5)(x-2)$
	$x = 5$, $x = 2$

Scoring:

1 pt: solving when $x - 5 \geq 0$

1 pt solving when $x - 5 \leq 0$

1 pt: eliminating incorrect answers

28) $|2x-6| = mx-4$

If $x=\frac{5}{2}$ is a solution

then $|2(\frac{5}{2})-6| = m(\frac{5}{2})-4$

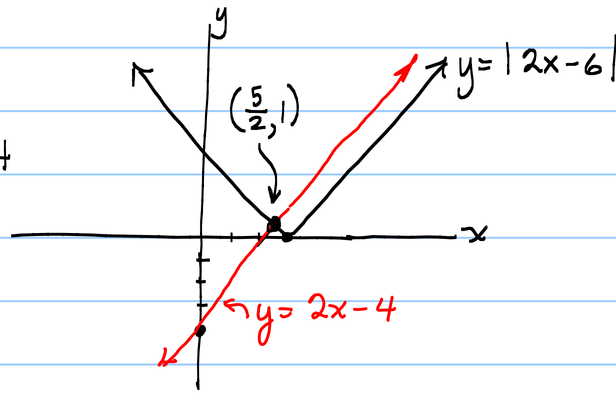
$|5-6| = \frac{5}{2}m - 4$

$1 = \frac{5}{2}m - 4$

$5 = \frac{5}{2}m$

$10 = 5m$

$m = 2$



Scoring:

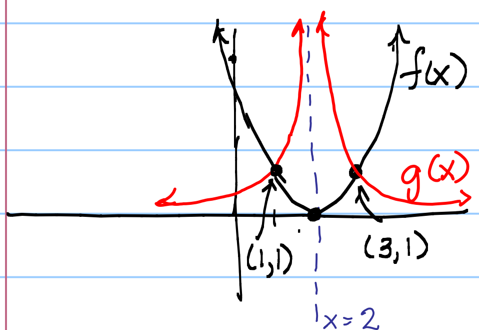
1pt: Substituting $x=\frac{5}{2}$

1pt: Solving for m

1pt: Illustration of solution.

29) $f(x) = x^2-4x+4 = (x-2)^2$

$g(x) = \frac{1}{f(x)} = \frac{1}{x^2-4x+4} = \frac{1}{(x-2)^2}$



Scoring:

1pt: Sketch of $f(x)$

$\frac{1}{2}$pt: Clear indication of vertical asymptote for $g(x)$

$\frac{1}{2}$pt: Points shared by $f(x)$ and $g(x)$

1pt: Sketch of $g(x)$

30)

x	g(x)
-2	1/8
-1	1/5
0	1/4
1	1/5

x	f(x)
-2	8
-1	5
0	4
1	5

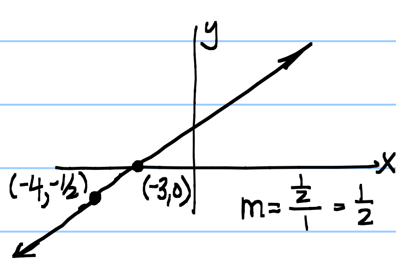
$f(x) = x^2+4$

Scoring

1pt: values of $f(x)$

1pt: equation of $f(x)$

31)



$m = \frac{\frac{1}{2}}{1} = \frac{1}{2}$

$f(x) = \frac{1}{2}x + b$

$(-3,0)$: $0 = \frac{1}{2}(-3) + b$

$\frac{3}{2} = b$

$\therefore f(x) = \frac{1}{2}x + \frac{3}{2}$

Scoring:

1pt: point $(-3,0)$ on $f(x)$

1pt: point $(-4,-\frac{1}{2})$ on $f(x)$

1pt: linear graph

1pt: equation of $f(x)$